YASHWANT CLASSES

Head Office: Govind Vihar Tower, Behind Vaishali Cinema, Badlapur (W)

Date : 28-06-2022
Time : 00:36:00
Marks : 60

TEST ID: 114
PHYSICS

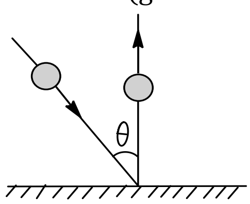
1.ROTATIONAL DYNAMICS ,7.SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Single Correct Answer Type

1. A ball of mass m moving with a velocity u collides head on with another ball of mass m initially at rest. If the coefficient of restitution be e then the ratio of the final and initial velocities of the first ball is
   a) $\frac{1-e}{1+e}$ b) $\frac{1+e}{1-e}$ c) $\frac{1+e}{2}$ d) $\frac{1-e}{2}$
2. A mass m is moving with a constant velocity along a line parallel to x-axis. Its angular momentum with respect to origin an z-axis is
   a) Zero b) Remains constant
   c) Goes on increasing d) Goes on decreasing
3. Particles of masses $m, 2m, 3m, \dots\dots, nm$ grams are placed on the same line at distances $l, 2l, 3l, \dots\dots, nl\ cm$ from a fixed point. The distance of centre of mass of the particles from the fixed point in centimeters is
   a) $\frac{(2n+1)l}{3}$ b) $\frac{l}{n+1}$
   c) $\frac{n(n^2+1)l}{2}$ d) $\frac{2l}{n(n^2+1)}$
4. A bullet of mass m leaves a gun of mass M kept on a smooth horizontal surface. If the speed of the bullet relative to the gun is v, the recoil speed of the gun will be
   a) $\frac{m}{M}v$ b) $\frac{m}{M+m}, v$
   c) $\frac{m}{M+m}v$ d) $\frac{M}{m}v$
5. A disc is rolling on the inclined plane, what is the ration of its rotational KE to the total KE?
   a) 1:3 b) 3:1 c) 1:2 d) 2:1
6. A rod of length l is hinged at one end and kept horizontal. It is allowed to fall. The velocity of the other end of the rod is
   a) $\sqrt{3gl}$ b) $\sqrt{2gl}$
   c) $2\ Ml^2$ d) None of these
7. The radius of gyration of a thin uniform circular disc (of radius R) about an axis passing through its centre and lying in its plane is
   a) R b) $\frac{R}{\sqrt{2}}$ c) $\frac{R}{4}$ d) $\frac{R}{2}$
8. A particle of mass m collides with another stationary particle of mass M. If the particle m stops just after collision, the coefficient of restitution of collision is equal to
   a) 1 b) $\frac{m}{M}$
   c) $\frac{M-m}{M+m}$ d) $\frac{m}{M+m}$
9. A particle of mass m moving with a velocity $(3\hat{i}+2\hat{j})\text{ms}^{-1}$ collides with a stationary body mass M and finally moves with a velocity $(-2\hat{i}+\hat{j})\text{ms}^{-1}$. If $\frac{m}{M}=\frac{1}{13}$, then
   a) The impulse received by each is , $\text{m}(5\hat{i}+\hat{j})$
   b) The velocity of the M is $\frac{1}{13}(5\hat{i}+\hat{j})$
   c) The coefficient of restitutions $\frac{11}{7}$
   d) All the above are correct
10. Moment of inertia of a disc about an axis which is tangent and parallel to its plane is I. Then the moment of inertia of disc about a tangent, but perpendicular to its plane will be
    a) $\frac{3I}{4}$ b) $\frac{5I}{6}$ c) $\frac{3I}{2}$ d) $\frac{6I}{5}$

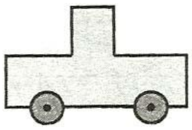
Integer Answer Type

11. A ball of mass 1 kg moving with a velocity of 5 m/s collides elastically with rough ground at an angle θ with the vertical as shown in Fig. What can be the minimum coefficient of friction if ball rebounds vertically after collision? (given $\tan\theta = 2$)

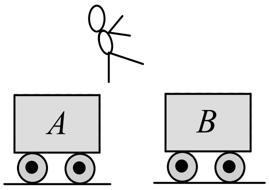


12. A uniform cylinder rests on a cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 CM in diameter and 10 CM in height, then what

is the minimum acceleration (in m/s^2) of the cart needed to cause the cylinder to tip over?

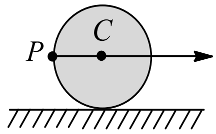


13. A child of mass 4 kg jumps from cart B to cart A and then immediately back to cart B. The mass of each cart is 20 kg and they are initially at rest. In both the cases the child jumps at 6 m/s relative to the cart. If the cart moves along the same line with negligible friction with the final velocities of V_B and V_A, respectively, find the ratio of $6v_B$ and $5v_A$



14. A ball of mass m makes head-on elastic collision with a ball of mass nm which is initially at rest. Show that the fractional transfer of energy by the first ball is $4n/(1+n)^2$. Deduce the value of n for which the transfer is maximum

15. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is given by $\sin^{-1}(\sqrt{2}/n)$.What is the value of n?



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1.ROTATIONAL DYNAMICS ,7.SYSTEM OF PARTICLES AND ROTATIONAL MOTION

: ANSWER KEY :

1)	**d**	**2)**	**b**	**3)**	**a**	**4)**	**b**
5)	**a**	**6)**	**a**	**7)**	**d**	**8)**	**b**
9)	**d**	**10)**	**d**	**11)**	**1**	**12)**	**4**
13)	**1**	**14)**	**1**	**15)**	**2**		

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1.ROTATIONAL DYNAMICS ,7.SYSTEM OF PARTICLES AND ROTATIONAL MOTION

: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(d)**

Here $m_1 = m_2 = m, u_1 = u$ and $u_2 = 0$

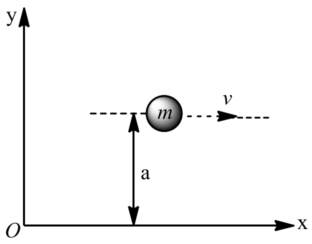
$$\therefore \quad v_1 = u_1\frac{(m_1 - em_2)}{(m_1 + m_2)} + u_2\frac{(1+e)m_2}{(m_1 + m_2)}$$

$$= \frac{u(1-e)}{2}$$

$$\Rightarrow \quad \frac{v_1}{u} = \left(\frac{1-e}{2}\right)$$

2 **(b)**

Angular moment of particle w.r.t., origin =linear momentum×perpendicular distance of line of action of linear momentum from origin



$$= mv \times a = mva = \text{constant}$$

3 **(a)**

$$x_{CM} = \frac{m_1x_1 + m_2 + x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$= \frac{ml + 2m.2l + 3m.3l + \cdots}{m + 2m + 3m + \cdots}$$

$$= \frac{ml(1 + 4 + 9 + \cdots)}{m(1 + 2 + 3 + \cdots)} = \frac{\frac{l\,n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}$$

$$= \frac{l(2n+1)}{3}$$

4 **(b)**

Speed of the bullet relative to ground $\vec{v}_b = \vec{v} + \vec{v}_r$, where v_r is recoil velocity of gun. Now for gun-bullet system applying the conservation law of momentum, we get

$m\vec{v}_b + M\vec{v}_r = 0$ or $m(\vec{v} + \vec{v}_r) + M\vec{v}_r = 0$

$\Rightarrow \; \vec{v}_r = -\frac{m\vec{v}}{m+M}$ or $v_r = \frac{mv}{m+M}$

5 **(a)**

The rotational kinetic energy of the disc is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}MR^2\omega^2$$

The translational kinetic energy is

$$K_{\text{trans}} = \frac{1}{2}Mv_{\text{CM}}^2$$

where v_{CM} is the linear velocity of its centre of mass.

Now, $v_{\text{CM}} = R\omega$

Therefore, $K_{\text{trans}} = \frac{1}{2}MR^2\omega^2$

Thus, $K_{\text{total}} = \frac{1}{4}MR^2\omega^2 + \frac{1}{2}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2$

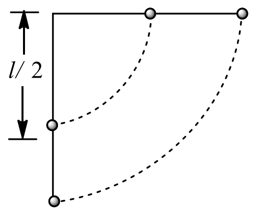
$$\therefore \quad \frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{4}MR^2\omega^2}{\frac{3}{4}MR^2\omega^2} = \frac{1}{3}$$

6 **(a)**

As the mass is concentrated at the centre of the rod, therefore,

$$mg \times \frac{l}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

or $l^2\omega^2 = 3gl$



Velocity of other end of the rod

$v = l\omega = \sqrt{3gl}$

8 **(b)**

As net horizontal force acting on the system is zero, hence momentum must remain conserved. Hence

$$mu + 0 = 0 + mv_2 \Rightarrow v_2 = \frac{mu}{M}$$

As per definition,

$$e = -\frac{(v_1 - v_2)}{(u_2 - u_1)} = \frac{v_2 - 0}{o - u} = \frac{v_2}{u} = \frac{\frac{mu}{M}}{u} = \frac{m}{M}$$

9 **(d)**

(a) Impulsive received by m

$\vec{J} = m(\vec{v}_f - \vec{v}_i)$

$= m(-2\hat{i} + \hat{j} - 3\hat{i} - 2\hat{j})$

$= m(-5\hat{i} - \hat{j})$

And impulse received by M

$= -\vec{J} = m(5\hat{i} + \hat{j})$

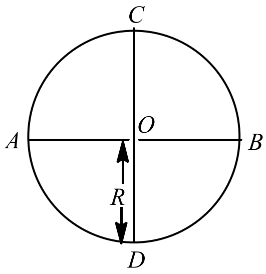
(b) $mv = m(5\hat{i} + \hat{j})$

Or $v = \frac{m}{M}(5\hat{i} + \hat{j}) = \frac{1}{13}(5\hat{i} + \hat{j})$

(c) $e =$ (relative velocity of separation/relative velocity of approach) in the direction of $-\vec{J} =$ 11/17

10 **(d)**

The moment of inertia of the disc about an axis parallel to its plane is



$$I_t = I_d + MR^2$$

$$\Rightarrow \quad I = \frac{1}{4}MR^2 + MR^2$$

$$= \frac{5}{4}MR^2$$

or $\quad MR^2 = \frac{4I}{5}$

Now, moment of inertia about a tangent perpendicular to its plane is

$$I' = \frac{3}{2}MR^2 = \frac{3}{2} \times \frac{4}{5}I = \frac{6}{5}I$$

Integer Answer Type

11 **(1)**

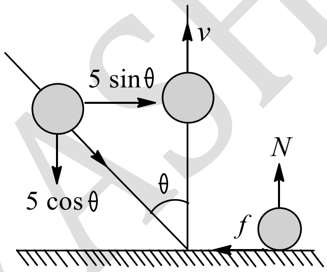
From impulse–momentum theorem,

$\int N\,dt = m(v + 5\cos\theta)$ (i)

$\int f\,dt = m5\sin\theta$

$\mu \int N\,dt = m5\sin\theta$ (ii)

$\Rightarrow \mu m(v + 5\cos\theta) = m5\sin\theta$



According to Newton's law of restitution,

$v = e\,5\cos\theta$

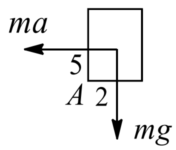
Solve to get $\mu = 1$

12 **(4)**

Maximum acceleration of the cart so that the cylinder does not slip:

$a_m = \mu g = 0.5 \times 10 = 5\ \text{m/s}^2$

For tipping over. Let acceleration of the cart be a



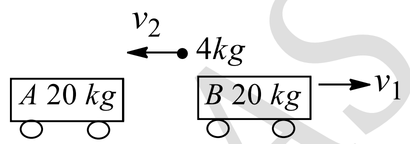
Considering torque about A, we get

$ma \times 5 \geq mg \times 2 \Rightarrow 2g/5 = 4\ \text{m/s}^2$

13 **(1)**

All the velocities shown in diagrams are w.r.t. ground

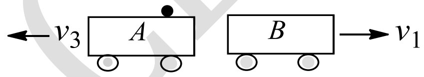
After first jump:



$20v_1 = 4v_2$ and $v_1 + v_2 = 6$ (given)

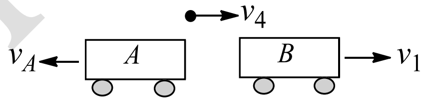
Solve to get $v_1 = 1\ \text{m/s}, v_2 = 5\ \text{m/s}$

When child arrives on A:



$(20 + 4)v_3 = 4v_2 \Rightarrow v_3 = 5/6\ \text{m/s}$

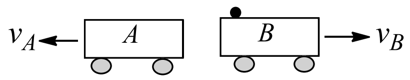
After the second jump:



$v_4 + v_A = 6, 24v_3 = 20v_A - 4v_4$

Solve to get $v_A = \frac{11}{6}\text{m/s}, v_4 = \frac{25}{6}\text{m/s}$

When child arrives on B:



$24v_B = 4v_4 + 20v_1$

$$\Rightarrow 24v_B = 4\left(\frac{25}{6}\right) + 20 \times 1 \Rightarrow v_B = \frac{55}{36}\ \text{m/s}$$

Now $\frac{6v_B}{5v_A} = \frac{6\times55\times6}{36\times5\times11} = 1$

14 **(1)**

Let u be the initial velocity of the ball of mass m. Then

$mu = mv_1 + nmv_2 \Rightarrow v_1 + nv_2 = u$ (i)

For elastic collision, Newton's experimental formula is $(u_2 = 0)$

$v_1 - v_2 = -(u_1 - u_2) \Rightarrow v_1 - v_2 = -u$ (ii)

Solving Eqs. (i) and (ii), $v_1 = \frac{1-n}{1+n}u$

Fractional loss in KE (= fractional transfer of KE)

$$f = \frac{K_i - K_f}{K_i} - \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mu^2} = 1 - \left(\frac{v_1}{u}\right)^2$$

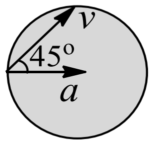
$f = 1 - \left(\frac{1-n}{1+n}\right)^2 = \frac{4n}{(n+1)^2}$

The transfer of energy is maximum when $f = 1$ or 100%

$\frac{4n}{(n+1)^2} = 1 \Rightarrow n = 1$

This is, the transfer of energy is maximum when the mass ratio is unity

15 **(2)**



The required angle is 45°, so $\frac{\sqrt{2}}{n} = \frac{1}{\sqrt{2}} \Rightarrow n = 2$